On the Determination of the Personal Equation of Lunar Observers.
By E. Neison.

In compliance with the request in Mr. Dunkin's Note in the *Monthly Notices* for January 1880 (page 166), I give a more detailed account of the method employed by me for determining the personal equations given in my Note in the previous number of the *Monthly Notices.*

Let the different observers be denoted by the Roman numerals I, II, III, ... and their personal equations by e', e'', e''', ... respectively, and as these personal equations are assumed to be liable to variations from year to year, let their values for any year $1860+k$ be denoted by e_k', e_k'', e_k''', . . . respectively.

Next let H_k denote the mean error for the year $1860 + k$ of the theory embodied in Hansen's lunar tables, and let the actual error at any epoch t be denoted by $H_k + h_t$ so that the quantity h_t will be one which quickly changes in sign and magnitude as the time varies. Further, let a_t denote the accidental error involved in the observation made at the epoch t. Then any observation made during the year $1860 + k$ at the epoch t by the observer I will differ from the tables by the quantity

$$H_k + h_t + a_t + e_k'.$$

Consider the mean of a number n of such observations

$$H_k + \frac{1}{n} \Sigma_n h_t + \frac{1}{n} \Sigma_n a_t + e_k'.$$

The quantities denoted by a_t being purely accidental and as likely to be positive as negative, the mean of any number n of these quantities will tend to become smaller as n, their number, increases. The mean may therefore be denoted by $an^{-\frac{1}{2}}$. Similarly, if the observations be impartially distributed over the period, the quantities denoted by h_t will also be of opposite signs and will tend to neutralise each other, so that the mean of n of these quantities will tend to become smaller as n, their number, increases, and may be denoted by $hn^{-\frac{1}{2}}$. If the observations be not impartially distributed, but there be a tendency for them to occur at some particular part of the lunation or year, then they will not so tend to destroy each other, and the mean will not necessarily decrease with an increase in the number of observations. In general, however, there is little tendency to such a systematic error, and if the tabular errors in the parallactic equation, apparent annual equation, and variation be corrected, there will be no such tendency.

From these considerations the mean of n observations may be written in the form

$$H_k + \frac{a+h}{\sqrt{n}} + \mathrm{E},$$

if E be put for the effect of the personal equations.

Suppose during any year 1860 + k there were n observations made, n' by observer I, n'' by observer II, n''' by observer III, and n^{iv} by observer IV. Then the mean error for the year would be

$$H_k + \frac{a+h}{\sqrt{n}} + \frac{n'}{n}e_k' + \frac{n''}{n}e_k'' + \frac{n'''}{n}e_k''' + \frac{n^{iv}}{n}e_k^{iv}. \tag{1}$$

Similarly, the mean of the n' observations made by the observer I would be

$$H + \frac{a+h}{\sqrt{n'}} + e_k'.$$

Therefore, calling the difference between these two the apparent personal error of the observer, its value is

$$E_k' = e_k' - \left(\frac{n'}{n}e_k' + \frac{n''}{n}e_k'' + \frac{n'''}{n}e_k''' + \frac{n^{iv}}{n}e_k^{iv}\right) + (a+h)\sqrt{\frac{n-n'}{nn'}}. \tag{2}$$

For each observer there will be a similar equation, so that there will be four equations to determine the four unknown quantities e_k', e_k'', e_k''', e_k^{iv}. If these equations were all independent it would be possible to completely determine each of these quantities in terms of the four known quantities E_k', E_k'', E_k''', E_k^{iv}, and the one unknown factor $(a+h)$, the sum of the mean values of the accidental errors of observation and the outstanding errors of theory. Owing to the presence of this unknown factor, it would be impossible to determine the actual personal equations with absolute accuracy.

One of these four equations, however, will be found to depend on the three others, owing to its having been used to eliminate H_k from the system. To render it possible, therefore, to determine the values of the personal equations, some arbitrary condition must be assumed, so as to obtain a fourth equation. This is equivalent to assuming some standard to which to refer the various observations, as, not knowing the absolute error of the tables, it is necessary to determine this error from the observations by referring them to some standard which is supposed to give the true place of the Moon. This condition I have taken to be that

$$e_k' + e_k'' + e_k''' + e_k^{iv} = 0. \tag{3}$$

Now, as the observations are supposed nearly equally divided amongst the four observers, it may be assumed that

$$n' = \frac{1}{4}n - m',$$

$$n'' = \frac{1}{4}n - m'',$$

$$n''' = \frac{1}{4}n - m''',$$

$$n^{iv} = \frac{1}{4}n - m^{iv},$$

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when m', m'', n''', m^{iv}, will be four small integers, either positive or negative, and usually less than one-twentieth of the value of n. Substituting these values, then to determine e_k', e_k'', e_k''', e_k^{iv}, there will be the four independent equations

$$\left.\begin{aligned} E_k' &= e_k' + \frac{1}{n}\left\{ m'e_k' + m''e_k'' + m'''e_k''' + m^{iv}e_k^{iv} \right\} + (a+h)\sqrt{\frac{n-n'}{nn'}}, \\ &\;\;\vdots \\ E_k^{iv} &= e_k^{iv} + \frac{1}{n}\left\{ m'e_k' + m''e_k'' + m'''e_k''' + m^{iv}e_k^{iv} \right\} + (a+h)\sqrt{\frac{n-n^{iv}}{nn^{iv}}}. \end{aligned}\right\} \quad (4)$$

In this manner it is possible to completely determine e_k' . . . e_k^{iv} in terms of known quantities and the small unknown factor $(a+h)$, this last factor rendering their exact values slightly uncertain.

Suppose in this manner the values of e' . . . . are obtained for several successive years, and that they can be represented by

$$\begin{array}{lcl} e_k' = e_o' + \Delta e_k', & \quad . \quad . \quad . \quad . \quad & e_k^{iv} = e_o^{iv} + \Delta e_k^{iv}, \\ e_{k+1}' = e_o' + \Delta e_k', & & e_{k+1}^{iv} = e_o^{iv} + \Delta e_{k+1}^{iv}. \\ \vdots & & \vdots \\ \vdots & . \quad . \quad . \quad . & \vdots \end{array}$$

If the quantities denoted by $\Delta e_k'$ . . . Δe_k^{iv} . . . . are small quantities, much smaller than the probable error of e_o' . . . and irregular in sign and value, it is obvious that each of these may be fairly assigned to the effect of the small error introduced by the small unknown quantity depending on $(a+h)$ and arising from the outstanding errors of observation and theory. For if there were any real variation in the personal equations, then these quantities $\Delta e_k'$ . . . Δe_k^{iv} . . . ought to reveal its existence by systematic variation in value, by regularly increasing and decreasing. If nothing of this kind occurs, then it may be fairly assumed that the personal equations have remained sensibly constant year after year.

Now, in the cases discussed in my paper this is actually the case. Thus, for instance, for the observers distinguished as III and IV the personal equations between 1863 and 1869 are

$$\begin{array}{ll} & \quad\;\; \text{s} \qquad\;\; \text{s} \\ e'''_3 = +{\cdot}057 - {\cdot}002, & e^{iv}_3 = -{\cdot}082 - {\cdot}014, \\ e'''_4 = +{\cdot}057 + {\cdot}006, & e^{iv}_4 = -{\cdot}082 + {\cdot}027, \\ e'''_5 = +{\cdot}057 - {\cdot}004, & e^{iv}_5 = -{\cdot}082 - {\cdot}008, \\ e'''_6 = +{\cdot}057 - {\cdot}010, & e^{iv}_6 = -{\cdot}082 - {\cdot}009, \\ e'''_7 = +{\cdot}057 + {\cdot}013, & e^{iv}_7 = -{\cdot}082 - {\cdot}008, \\ e'''_8 = +{\cdot}057 + {\cdot}004, & e^{iv}_8 = -{\cdot}082 - {\cdot}003, \\ e'''_9 = +{\cdot}057 - {\cdot}008, & e^{iv}_9 = -{\cdot}082 + {\cdot}006. \end{array}$$

In finding then the personal equations of these observers, it

may justly be assumed that their personal equations remained constant throughout this period, and all seven years may be grouped together to determine them. Moreover, as the personal equations have remained constant, the standard to which they have been referred must have remained constant, as it is derived from the condition

$$e_o' + e_o'' + e_o''' + e_o^{\text{iv}} = 0, \tag{5}$$

As long as the observations are made by the same four observers, the preceding method can be adopted without alteration. Now, suppose in the year $1860 + j$, one of the four observers, say I, is replaced by a new observer, say V; then throughout the quantity e' will be replaced by e^{v}. The method of determining the personal equations will remain unaltered, but they will now be referred to the standard derived from the condition

$$e_j'' + e_j''' + e_j^{\text{iv}} + e_j^{\text{v}} = 0, \tag{6}$$

and not, as before, from the condition that

$$e_o' + e_o'' + e_o''' + e_o^{\text{iv}} = 0.$$

It will therefore be necessary to take into account the effect of this change in the standard to which the observations are referred.

Now, suppose these two standards differ by a small quantity c, and that (e_j'') (e_j''') (e_j^{iv}) . . . represent the personal equations as determined by the new standard (6); then each of these will be greater than its true value referred to the original standard by the quantity c. Therefore c can be determined by the equation

$$e_o'' + e_j''' + e_o''' = (e'') + (e_j'') + (e_j^{\text{iv}}) + 3c,$$

$$c = \frac{1}{3}\,\frac{(e_o'') + e_o''' + e_o^{\text{iv}}}{(e_j'') + (e_j''') + (e_j^{\text{iv}})}, \tag{7}$$

provided it be assumed that no change has occurred during the year in the personal equations of these three observers. This assumption is reasonable enough. If they have been constant for several years previous to this, it is unlikely that they should change at this very time that a new observer is introduced, as the mere change of observers cannot give rise to any such variation. It is of course possible that one observer may have chanced to change his personal equation at this very epoch; but, even if this be so, only one-third of this change will be thrown on the determination of the difference between the two standards: this risk is unavoidable. Even, however, if one observer had changed his personal equation by the small increment x, then c would undergo a fictitious increase of $\frac{1}{3}\,x$; and when the personal equations came to be compared with those of previous years, whilst two would seem to have decreased by $\frac{1}{3}\,x$, the third, which had really varied, would seem to have increased by $\frac{2}{3}\,x$.

If, therefore, only one had varied, this would serve to indicate which with some degree of probability. For it is much more probable that one should have varied than that two should have done so and in the same direction.

It has been already shown that it is possible to determine with some certainty whether the personal errors of the observers have changed during any group of years $k, k+1, \ldots j-1$, through which period the observers have remained the same. Suppose a break to occur at the beginning of the year j, through one observer being replaced by another, and that this fresh arrangement remains unaltered for the group of years $j, j+1, \ldots l-1$; then it can be ascertained in exactly the same manner whether the personal equations of the observers have remained unchanged for the period between the years j and l. Suppose during both these groups they remain unaltered, then the difference between the two standards can be determined with considerable approximation by the comparison of the mean results for each group of years in the manner already detailed. The difference—denoted by a in my paper—between these two standards having been determined, it is possible to reduce the personal equations for the entire period covered by both groups to any standard which may be assumed.

The same process may then be extended to a third group of years commencing with the year l in which yet another old observer is replaced by a new observer, and in this manner a third set of personal equations determined and reduced to the same standard as before.

It is in this manner that the four quantities a_1, a_2, a_3, a_4, were determined which were given in my former paper, and by means of which I was able to determine the personal equations of the different observers, *without* being obliged to assume either that these personal equations remained constant, or even that the personal equations of any one observer must be assumed to remain constant. It so happens that I find the personal equations of the older observers to be constant, so that their effect can be accurately computed for the years between 1862 and 1873. The later observers seem to have personal equations liable to variation, or through less experience are liable to greater accidental errors of observation. In my former paper, having practically only two groups, each of two years' duration, from which to determine two new personal equations, the results were less accurate than could be desired. When, however, the observations for 1877, 1878 and 1879 are available it will be possible to arrive at more definite results on this most important matter.

I think the detailed account given above will remove any doubts which Mr. Dunkin may have entertained as to the soundness of the method by which were determined the personal equations given in my paper. It is obvious that, admitting that Mr. Thackeray's personal equation may be liable to vary or its value may be founded on too few observations, these things can have

no influence in vitiating the results arrived at by me, as the results for the other observers do not depend in the least on the value assigned by me to Mr. Thackeray's personal equation. At the same time I would remark that in my paper I point out that the results for Mr. Downing, and more especially for Mr. Thackeray, rest on so few observations that they must be considered uncertain.

Mr. Dunkin, in the concluding paragraph of his Note, gives the results of his determination of the personal equations of Messrs. Lynn, Downing and Thackeray, as compared with Mr. Criswick for the three years 1877 and 1878, derived from data inaccessible to me. Assuming Mr. Criswick's error to have remained unaltered, these results transformed into my own standard show that the personal equations of Mr. Downing and Mr. Thackeray have changed since 1876. The correction to be applied to the mean of the observations of both limbs, in order to reduce Mr. Downing's and Mr. Thackeray's observations to my standard is given in the following table together with those derived from the data supplied by Mr. Dunkin.

	1875	1876	1877	1878
	s	s	s	s
Mr. Downing =	−0·086	−0·097	+0·033	−0·017
Mr. Thackeray =	−0·108	−0·130	−0·033	−0·098
Mean effect on year	−0·057	−0·054	+0·020	−0·020

The mean effect on the year is derived by referring the personal equations to the standard adopted by me and taking into account the number of observations made by each observer. The above results show that the change in the personal equations will produce an apparent diminution of the tabular error of the Moon amounting to about 1″·2 in 1877 and 0″·6 in 1878. Now, this is exactly what is shown by the observations, the apparent decrease in the tabular errors for these years being 1″·3 for 1877 and 0″·9 for 1878, taking into account the effect of the apparent decrease in the tabular error due to the change in the nature of the systematic errors of long and short period. So far, therefore, from the results given by Mr. Dunkin invalidating the conclusions arrived at in my paper they tend to strongly confirm them.

Supplementary Note on the Changes in the Error of Hansen's Lunar Tables. By W. T. Lynn, B.A.

This Note is simply a supplement to the paper printed at page 82 of the present volume (*Monthly Notices* for December 1879), giving the completion of the mean errors in longitude up to the end of last year, as deduced from the Altazimuth observations. And, at the close of my superintendence of that instru-